

Evolution of a coherent array of Bose-Einstein Condensates in a magnetic trap

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*We investigate the evolution process of the interference pattern for a coherent array of Bose-Einstein condensates in a magnetic trap after the optical lattices are switched off. It is shown that there is a decay and revival of the density oscillation for the condensates confined in the magnetic trap. We find that, due to the confinement of the magnetic trap, the interference effect is much stronger than that of the experiment induced by Pedri et al. (Phys. Rev. Lett, **87**, 220401), where the magnetic trap is switched off too. The interaction correction to the interference effect is also discussed for the density distribution of the central peak.*

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I. INTRODUCTION

The development of the technologies of the laser trapping and evaporative cooling has yielded intriguing Bose-Einstein condensates (BECs) [1], a state of matter in which many atoms are in the same quantum mechanical state. The realization of BECs has made remarkable theoretical and experimental advances on this exotic quantum system [2,3]. Recently, optical lattices are used to investigate further the unique character of the ultra-cold atoms [4]. The application of the optical lattices to the ultra-cold atoms is very promising, such as the quantum computing scheme proposed in [5,6]. Experimentally, the macroscopic quantum interference effect [7] and thermodynamic properties [8] of the BECs in the optical lattices have been investigated thoroughly. In addition, the superfluid and dissipative dynamics [9,10] of a BEC in the optical lattices are investigated in the experiments too. In particular, through the application of the optical lattices, the quantum phase transition from a superfluid to a Mott insulator in Bose-Einstein condensed gases is observed in a recent experiment [11].

Recently, the expansion of a coherent array of BECs is carried out to illustrate the interference effect in the experiment by Pedri *et al.* [12], when both the magnetic trap and optical lattices are switched off. In another experiment by Morsch *et al.* [13], the expansion of the condensates is investigated when only the magnetic trap is switched off. In this paper, we give a theoretical investigation on the expansion of the condensates when only the optical lattices are switched off. Due to the confinement of the harmonic potential, we find that there are several interested phenomena not encountered in the experiment by Pedri *et al.* [12] and Morsch *et al.* [13]. In the presence of the harmonic potential, the interference effect would be much stronger than the case when the magnetic trap is switched off too. In this situation, researches show that the interaction between atoms would give important correction, in contrast to the case investigated by Pedri *et al.* [12].

II. WAVE FUNCTION IN HARMONIC POTENTIAL AFTER OPTICAL LATTICES ARE SWITCHED OFF

In the experiment conducted by Pedri *et al.* [12], the external potential of the Bose gas is given by [12]:

$$V = \frac{1}{2}m(\omega_x^2 x^2 + \omega_\perp^2 (y^2 + z^2)) + sE_R \cos^2\left(\frac{2\pi x}{\lambda} + \frac{\pi}{2}\right), \quad (1)$$

The last term represents the external potential due to the presence of the optical lattices. In the above expression, ω_x and ω_\perp are the axial and radial frequencies of the harmonic potential, respectively. In addition, λ is the wavelength of the retroreflected laser beam, and sE_R denotes the depth of the optical lattices. For the optical lattices created by the retroreflected laser beam, the last term has a period $d = \lambda/2$. In other words, d can be regarded as the distance between two neighboring wells induced by the optical lattices. In this paper, the experimental parameters in [12] are used to calculate various physical quantities, thus it would be useful to give them here. The experimental parameters in [12] are $\omega_x = 2\pi \times 9$ Hz, $\omega_\perp = 2\pi \times 92$ Hz, $\lambda = 795$ nm, $E_R = 2\pi\hbar \times 3.6$ kHz, and $s = 5$.

Due to the presence of the optical lattices, there are an array of condensates formed in the combined potential, when the temperature is lower than the critical temperature. In this work, we shall investigate the case of the strong tunneling between neighboring BECs, which holds in the experiment by Pedri *et al.* [12]. In this situation, all the condensates are fully coherent, and can be described by a single order parameter. To emphasis the role of the optical lattices, our researches are carried out mainly on the character of the coherent array of condensates in the x -direction.

In the presence of a magnetic trap, the number of atoms in each well should be different. Based on the analysis of the 3D model of the condensates in the combined potential [12], the ratio between the number of condensed atoms in k -th and central wells is given by $N_k/N_0 = (1 - k^2/k_M^2)^2$, where $2k_M + 1$ [12] represents the total number of condensates induced by the optical lattices. In this situation, using the Gaussian approximation in the x -direction for each well, the normalized wave function in coordinate space takes the form

$$\varphi_0(x) = A_n \sum_{k=-k_M}^{k_M} \left(1 - \frac{k^2}{k_M^2}\right) \exp[-(x - kd)^2/2\sigma^2], \quad (2)$$

where σ denotes the width of the condensate in each well. It can be calculated by numerical minimization of the energy of the condensates [12]. In the following calculations in this paper, $\sigma = 0.25d$ [12] is used for $s = 5$. In the above expression, the normalized constant A_n takes the form

$$A_n = \frac{1}{\sqrt{(16k_M^4 - 1)/15k_M^3} \pi^{1/4} \sigma^{1/2}}. \quad (3)$$

It is well known that once the wave function at an initial time is known, the wave function at a later time can be obtained through the following integral equation [14]:

$$\varphi_0(x, t) = \int_{-\infty}^{\infty} K(x, t; y, t = 0) \varphi_0(y, t = 0) dy, \quad (4)$$

where $\varphi_0(y, t = 0)$ is the wave function at the initial time $t = 0$ which is given by Eq. (2), and $K(x, t; y, t = 0)$ is the well known propagator. For the atoms in the harmonic potential, the propagator can be expressed as [14]:

$$K(x, t; y, t = 0) = \left[\frac{m\omega_x}{2\pi i \hbar \sin \omega_x t} \right]^{1/2} \exp \left\{ \frac{im\omega_x}{2\hbar \sin \omega_x t} [(x^2 + y^2) \cos \omega_x t - 2xy] \right\}. \quad (5)$$

From the formulas (2), (4) and (5), after a straightforward calculation, one obtains the analytical result of the wave function confined in the magnetic trap

$$\begin{aligned} \varphi_0(x, t) = A_n \sqrt{\frac{1}{\sin \omega_x t (\text{ctg} \omega_x t + i\gamma)}} \sum_{k=-k_M}^{k_M} \left(1 - \frac{k^2}{k_M^2}\right) \exp \left[-\frac{(kd \cos \omega_x t - x)^2}{2\sigma^2 \sin^2 \omega_x t (\text{ctg}^2 \omega_x t + \gamma^2)} \right] \times \\ \exp \left[-\frac{i(kd \cos \omega_x t - x)^2 \text{ctg} \omega_x t}{2\gamma \sigma^2 \sin^2 \omega_x t (\text{ctg}^2 \omega_x t + \gamma^2)} \right] \exp \left[\frac{i(x^2 \cos \omega_x t + k^2 d^2 \cos \omega_x t - 2xkd)}{2\gamma \sigma^2 \sin \omega_x t} \right], \end{aligned} \quad (6)$$

where we have introduced a dimensionless parameter $\gamma = \hbar/m\omega_x\sigma^2$. Assuming N denotes the total number of particles in the condensates, the density distribution in x -direction is $n(x, t) = N |\varphi_0(x, t)|^2$.

III. PERIODICITY OF THE DENSITY DISTRIBUTION AND THE MOTION OF $N = \pm 1$ PEAK

Due to the confinement of the harmonic potential, the density distribution in x -direction should exhibit a periodic character. From Eq. (6), it is easy to find that the period of the density distribution $n(x, t)$ is determined by $\omega_x T = \pi$. For the experiment in [12], $\omega_x = 2\pi \times 9$ Hz, this means that the period of the density distribution is given by 500/9 ms.

In Fig. 1, displayed is $n(x = 0, t)$ when only the optical lattices are switched off. The periodicity of the density is clearly shown in the figure and in agreement with the analytical result given by $T = \pi/\omega_x$. We see that the density at $x = 0$ reaches a maximum value at time $t_m = (2m - 1)\pi/2\omega_x$, with m positive integer. At time t_m , the wave function takes the form

$$\varphi_0(x, t_m) = A_n \sqrt{\frac{1}{i\gamma}} \sum_{k=-k_M}^{k_M} \left(1 - \frac{k^2}{k_M^2}\right) \exp \left[-\frac{x^2}{2\sigma^2\gamma^2} - i\frac{xkd}{\gamma\sigma^2} \right]. \quad (7)$$

The maximum density at $x = 0$ is then

$$n(x = 0, t_m) = \frac{NA_n^2}{\gamma} \left[\sum_{k=-k_M}^{k_M} \left(1 - \frac{k^2}{k_M^2}\right) \right]^2. \quad (8)$$

In the case of $k_M \gg 1$, the above expression can be approximated as:

$$n(x = 0, t_m) \approx N\alpha_{x-ideal}^2, \quad (9)$$

where $\alpha_{x-ideal}^2$ is given by

$$\alpha_{x-ideal}^2 = \frac{5k_M m \omega_x \sigma}{3\pi^{1/2} \hbar}. \quad (10)$$

In the experiments, the depth of the optical lattices can be changed through the variation of the parameter s . From [12], we know $\sigma \propto 1/s^{1/4}$ and $k_M \propto 1/\sigma^{1/5}$. Therefore, $n(x = 0, t_m) \propto 1/s^{1/5}$. This shows that in the non-interacting model, the maximum density at $x = 0$ decreases with the increase of the depth of the optical lattices.

In Figures 2 (a)-(d), we show the evolution of the density distribution of the condensates confined in the magnetic trap, after the optical lattices are switched off. The density distributions are shown at $t = 0, 0.1\pi/\omega_x, 0.3\pi/\omega_x$, and $0.5\pi/\omega_x$ in these figures. The motion of the $n = \pm 1$ peaks is clearly shown in these figures. The oscillation motion of the $n = \pm 1$ peaks is due to the confinement of the harmonic potential. In fact, the motion of the $n = \pm 1$ peaks can be described very well using the classical harmonic motion. Using the classical harmonic motion, the motion of the $n = \pm 1$ peaks is determined by the following expression:

$$x_{n=\pm 1}(t) = \pm \frac{2\pi\hbar}{m\omega_x d} \cos\left(\omega_x t - \frac{\pi}{2}\right). \quad (11)$$

When obtaining the above formula, we have used the fact that the momentum distribution is characterized by sharp peaks at the values $p_x = n2\pi\hbar/d$ [12]. The solid line in Fig. 3 shows the harmonic motion of the $n = 1$ peak using the above formula, while the circles show the result given by Eq. (6). We see that the classical harmonic motion agrees quite well with the result given by Eq. (6). In a sense, Eq. (11) describes the motion of the center of mass of the $n = \pm 1$ peak. Thus, we anticipate that the interaction between atoms will not change the motion of the $n = \pm 1$, although it will affect the density and width of the $n = \pm 1$ peak.

From Fig. 2(d) we see that at time t_m the density distribution in x -direction exhibits a very sharp peak at the center of the magnetic trap. The maximum density of the central peak is given by Eq. (9). As a comparison, assume there are N atoms confined in an identical magnetic trap, but there are no optical lattices to induce the interference effect. In this situation, in x -direction, the density distribution at $x = 0$ is given by

$$n_{mag}(x = 0) = \frac{\pi\mu_{mag}^2}{gm\omega_\perp^2}, \quad (12)$$

where μ_{mag} is the chemical potential of the Bose gas confined in the magnetic trap. The ratio between $n(x = 0, t_m)$ and $n_{mag}(x = 0)$ is then

$$\frac{n(x = 0, t_m)}{n_{mag}(x = 0)} = \frac{Ngm\omega_\perp^2\alpha_{x-ideal}^2}{\pi\mu_{mag}^2}. \quad (13)$$

For the experimental parameters in [12], $n(x = 0, t_m)/n_{mag}(x = 0) = 29.6$. This shows clearly that there is a very strong interference effect for the case considered here.

If both the magnetic trap and optical lattices are switched off, there is also a sort of interference effect. The maximum value of the density distribution in this case is given by

$$n_{bs}(x = 0) = \frac{N_0}{\sqrt{\pi}\sigma}. \quad (14)$$

From Eq. (14), $n_{bs}(x=0) \propto s^{1/4}$. In contrast to the case where only the optical lattices are switched off, $n_{bs}(x=0)$ decreases with the decrease of the depth of the optical lattices.

The ratio between $n_{bs}(x=0)$ and $n_{mag}(x=0)$ is then

$$\frac{n_{bs}(x=0)}{n_{mag}(x=0)} = \frac{N_0 g m \omega_{\perp}^2}{\pi^{3/2} \sigma \mu_{mag}^2}. \quad (15)$$

For the experimental parameters in [12], we have $n_{bs}(x=0)/n_{mag}(x=0) = 2.2$. From Eqs. (9) and (14), $n(x=0, t_m)/n_{bs}(x=0) = 13.7$. Therefore, when only the optical lattices are switched off, the interference effect would be much stronger than the case when the magnetic trap is switched off too. When only the optical lattices are switched off, at t_m the density of the central peak is very high, and we anticipate that in this situation the interaction between atoms would give important correction.

IV. DECAY AND REVIVAL OF THE DENSITY OSCILLATION

From Fig. 1, we see that there is a phenomenon of decay and revival of the density oscillation at $x=0$. We now turn to discuss this unique character. To proceed, it is useful to introduce an important time scale which determines when the interference between two neighboring condensates begins to occur. Before the magnetic trap and optical lattices are switched off, from the Gaussian approximation of the condensates in each well, the width of the condensates in each well is given by

$$\Delta x_0^2 = \frac{\int x^2 \exp[-x^2/\sigma^2] dx}{\int \exp[-x^2/\sigma^2] dx}. \quad (16)$$

It is easy to get $\Delta x_0 = \sigma/\sqrt{2}$ from the above formula. When the optical lattices are switched off, for time much smaller than π/ω_x , the condensates can be approximated as a free expansion and the width of the condensates would increase in this situation. Based on the analysis of the spreading of the wave packet, the width of each condensate is given by

$$\Delta x(t) = \Delta x_0 \sqrt{1 + \frac{\hbar^2 t^2}{m^2 \Delta x_0^4}}. \quad (17)$$

When $\Delta x(t) = d$, the condensates in neighboring wells begin to interfere with each other. By setting $\Delta x(t) = d$ in the above formula, we obtain a time scale t_w which determines when the interference between neighboring condensates begins to occur. From Eq. (17), it is easy to find that the following analytical result of t_w can give a rather well approximation

$$t_w = \frac{\sigma d m}{\sqrt{2} \hbar}. \quad (18)$$

From Eqs. (17) and (18), for $t > t_w$, one gets the following useful result:

$$\Delta x(t) = \frac{t}{t_w} d. \quad (19)$$

As illustrated in Fig. 1, the oscillation of the density at $x=0$ will cease ultimately when $t > k_M t_w$. However, when the time approaches π/ω_x , the density oscillation will reappear. Note that the time for the revival of the density oscillation is determined solely by the axial frequency of the harmonic potential. This shows that the confinement of the harmonic potential plays a crucial role for the revival phenomenon of the density oscillation. To verify further the decay and revival of the density oscillation, Fig. 4 shows the density at $x = k_M d/2$. Analogous decay and revival of the density oscillation are illustrated clearly in the figure. However, the oscillation of the density disappears at a longer time $t = 1.47 k_M t_w$, in comparison with the case at $x=0$.

In Fig. 5, we display the time of the disappearance of the density oscillation for different locations in the region $0 < t < \pi/\omega_x$. We can give a rather simple interpretation for this result. When the optical lattices are switched off, the width of the expanding condensates in each well will increase. For the location at $x=0$, there are more and more expanding BECs interfere at this point with the development of the time. This is the reason why the density at the point $x=0$ will oscillate intensely. When $t > k_M t_w$, however, all expanding BECs have participated in the

interference at the point $x = 0$. Therefore, the oscillation of the density at $x = 0$ will cease at time longer than $k_M t_w$. Generalizing this result, the time for the disappearance of the density oscillation for different locations is given by the following simple expression:

$$t = (k_M + x/d)t_w. \quad (20)$$

The solid line in Fig. 5 displays the above analytical result. We see from Fig. 5 that this simple expression agrees well with the result given by Eq. (6). Maybe the slight difference from the result given by Eq. (6) lies in the fact that we do not account for the effect of non-uniform atom distribution in each well when obtaining Eq. (20).

It is worth pointing out that the disappearance of the density oscillation do not mean the disappearance of the interference effect between the expanding condensates. When $0 < t < (2k_M + 1)t_w$, there are only several expanding condensates interfering with each other, and there are interference fringes (or density oscillation) in this situation. For $(2k_M + 1)t_w < t < \pi/\omega_x - (2k_M + 1)t_w$, however, all expanding condensates will interfere with each other, and this means the emergence of the diffraction fringes. In fact, $n = 0$ and $n = \pm 1$ peaks in Fig. 2 (a)-(d) should be regarded as the diffraction fringes, rather than the interference fringes. Note that the phenomena of diffraction and interference are basically equivalent. Different from the interference phenomenon, however, the diffraction phenomenon should be regarded as a consequence of interference from many coherent wave sources. In a sense, Eq. (20) gives the time for the emergence of the diffraction fringes, which means the disappearance of the density oscillation.

V. INTERACTION CORRECTION TO THE CENTRAL PEAK AT T_M

In the case of non-interacting model, we have shown that, at time t_m , there would be a sharp central peak in the magnetic trap. In this case, the interaction between atoms can not be simply omitted, in contrast to the case when the magnetic trap is switched off too. At time t_m , the central peak of the density in x -direction can be approximated as a Gaussian distribution. After the optical lattices are switched off, using Thomas-Fermi approximation in the radial direction, the square of the modulus of the 3D wave function at time t_m takes the form

$$|\varphi_0(x, r_\perp, t_m)|^2 = \alpha_x^2 \alpha_\perp^2 \exp\left[-\frac{2x^2}{R_x^2}\right] \left(1 - \frac{r_\perp^2}{R_\perp^2}\right), \quad (21)$$

where $R_\perp^2 = \sqrt{2\mu_0/m\omega_\perp^2}$ with $\mu_0 = m\omega_x^2 k_M^2 d^2/2$ [12]. In the above expression, $\alpha_x = (2/\pi)^{1/4}/\sqrt{R_x}$ and $\alpha_\perp = \sqrt{2/\pi}R_\perp^2$ are the normalized constants in the axial and radial directions, respectively. Obviously, $N\alpha_x^2$ represents the density $n(x=0, t_m)$ in the x -direction. Due to the repulsive interaction between atoms, we anticipate that $\alpha_x^2 < \alpha_{x-ideal}^2$.

Assume E_{int} and E_{kin} are the interaction energy and kinetic energy of the central peak at t_m , respectively. The interaction energy of the central peak is given by

$$E_{int} = \frac{gN^2}{2} \int |\varphi_0(x, r_\perp, t_m)|^4 dV = \frac{\sqrt{2}gN^2\alpha_x^2}{3\pi R_\perp^2}. \quad (22)$$

Assuming the total energy of the condensates is E_{all} , we have

$$E_{kin} + E_{int} + E_{ho} = E_{all}, \quad (23)$$

where E_{ho} is the potential energy of the condensates. For the central peak, E_{ho} can be omitted safely. In the case of non-interacting model, E_{all} will transform fully to the kinetic energy and potential energy of the condensates once the optical lattices are switched off. Due to the presence of the repulsive interaction between atoms, the maximum density of the central peak would be smaller than the result of the non-interacting model.

Note that the kinetic energy of the central peak can not be calculated through $N \int \frac{\hbar^2}{2m} \left(\nabla \sqrt{|\varphi_0(x, r_\perp, t_m)|^2}\right)^2 dV$, because the phase factor is different for different well (See Eq. (7)). From the uncertainty relation, assume $E_{kin} \propto 1/R_x^2 \propto \alpha_x^4$. In the presence of repulsive interaction, we have

$$E_{kin} = \frac{\alpha_x^4}{\alpha_{x-ideal}^4} E_{all}. \quad (24)$$

From Eqs. (22), (23) and (24), one obtains the following equation to determine α_x^2 :

$$\beta^2 + \theta\beta - 1 = 0, \quad (25)$$

where $\beta = (\alpha_x / \alpha_{x-ideal})^2$. The value of β reflects how the repulsive interaction between atoms reduces the density of the central peak. In the above expression, the dimensionless parameter $\theta = E_{int}(\alpha_x = \alpha_{x-ideal}) / E_{all}$. From Eq. (25), we have

$$\beta = \frac{-\theta + \sqrt{\theta^2 + 4}}{2}. \quad (26)$$

Now let us turn to discuss the total energy of the condensates which is necessary to calculate the value of β . The total energy of the condensates can be obtained through the sum of the energy of the condensates in each well before the optical lattices are switched off. Before the optical lattices are switched off, the normalized wave function in k -th well takes the form

$$\varphi_{0k}(x, r_\perp) = \varphi_{0k}(x) \varphi_{0k}(r_\perp) \quad (27)$$

where

$$\varphi_{0k}(x) = \left(\frac{1}{\sqrt{\pi}\sigma} \right)^{1/2} \exp \left[-\frac{(x - kd)^2}{2\sigma^2} \right], \quad (28)$$

and $\varphi_{0k}(r_\perp)$ is the normalized wave function in the radial direction. From Eq. (27), we have

$$E_{all} = \sum_{k=-k_M}^{k_M} \left\{ N_k \int \varphi_{0k}(x, r_\perp) \left[-\frac{\hbar^2}{2m} \nabla^2 + \frac{1}{2} m \omega_{xe}^2 x^2 + \frac{1}{2} m \omega_\perp^2 r_\perp^2 \right] \varphi_{0k}(x, r_\perp) dV \right. \\ \left. + \frac{g N_k^2}{2} \int |\varphi_{0k}(x, r_\perp)|^4 dV \right\}, \quad (29)$$

where ω_{xe} is the effective harmonic frequency in the x -direction of the well induced by the optical lattices. The last term in the above expression represents the interaction energy of the condensates in each well, and it is easy to verify that it can be omitted safely. In addition, the kinetic energy and potential energy in the radial direction can be also omitted because $\omega_{xe} \gg \omega_\perp$, and $\hbar\omega_{xe} \gg \mu_0$. In this case, one gets

$$E_{all} \approx N \left(\frac{\hbar^2}{4m\sigma^2} + \frac{1}{4} m \omega_{xe}^2 \sigma^2 \right). \quad (30)$$

For the experimental parameters in [12], from the formulas (22), (26), and (30), the calculation shows that $\beta = 0.80$. This shows clearly that the repulsive interaction between atoms would reduce the density of the central peak at t_m .

With the variation of the parameter s , the maximum density of the central peak can be also calculated based on the method given here. In the interacting model, the solid line in Fig. 6 shows the ratio between $n(x=0, t_m, s)$ and $n(x=0, t_m, s=5)$. The result of the non-interacting model is also shown in the figure.

VI. DISCUSSION AND CONCLUSION

In brief, the evolution process of the condensates is investigated after the optical lattices are switched off. We find that the density oscillation exhibits a phenomenon of decay and revival, based on the numerical result of the evolution of the density distribution. The decay of the density oscillation is interpreted as the emergence of the diffraction phenomenon, which is regarded as a consequence of interference from a lot of coherent expanding condensates. Due to the confinement of the harmonic potential, there is a periodic character of the density distribution, and it is this periodic character which leads to the revival of the density oscillation. In contrast to the condensates in the magnetic trap, there is no revival of the density oscillation, when both the magnetic and optical lattices are switched off. In addition, in the case of non-interacting model, it is shown that the maximum value of the density distribution at $x=0$ would be approximately 30 times larger than the case when there are no optical lattices to induce the interference effect.

It is shown here that the repulsive interaction between atoms has an effect of reducing the maximum density of the central peak. In a real experiment in future, maybe the experimental result of the maximum density of the central

peak would be smaller than the theoretical prediction given here, because it is possible that there is a loss of the total energy of the condensates during the process of the removal of the optical lattices. For the attractive interaction such as Li , the role of interaction would become very important when the atoms are confined by the combined potential. For example, when only the optical lattices are removed, based on the non-interacting model, the density at $x = 0$ would increase largely due to the interference and confinement of the magnetic trap. In addition, due to the attractive interaction between atoms, the density of the central peak would increase rapidly. In this situation, it is possible that the condensates would collapse and even explode in a subsequent time, in analogous with the dynamic process of the collapsing and exploding atoms [15] by switching the interaction from repulsive to attractive.

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FIGURE CAPTION

Fig. 1 Displayed is the density of the condensate at $x = 0$ vs time t , after the optical lattice is switched off. Here the density $n(x = 0, t)$ is in units of NA_n^2 . We can see clearly in the figure that there is a decay and revival of the density oscillation. In addition, there is a periodicity of the density due to the confinement of the magnetic trap.

Fig. 2 (a)-(d) show the evolution of the density distribution with time t , after the optical lattices are switched off. The density distributions are shown at $t = 0, 0.1\pi/\omega_x, 0.3\pi/\omega_x$, and $0.5\pi/\omega_x$. The emergence and motion of the $n = \pm 1$ peaks are clearly shown in these figures. Here the density distribution $n(x, t)$ is in units of NA_n^2 , while the location x is in units of d , *i.e.* the distance between two neighboring condensates.

Fig. 3 Displayed is the motion of the $n = 1$ peak, after the optical lattices are switched off. Here the location $x(t)_{n=1}$ of $n = 1$ peak is in units of d . The solid line is the result calculated from the classical harmonic motion given by Eq. (11). The squares show the motion of $n = 1$ peak obtained from the numerical result given by Eq. (6). We see that the classical harmonic motion agrees quite well with the numerical result.

Fig. 4 Displayed is the density of the condensate at $x = k_M d/2$ vs time t , after the optical lattices are switched off. Here the density $n(x, t)$ is in units of NA_n^2 . There is a phenomenon of the decay and revival of the density oscillation.

Fig. 5 Displayed is the time of the disappearance of the density oscillation for different locations. Here the location x is in units of d , while the time t is in units of the time scale t_w . The solid line is obtained from the formula (20), while the squares show the result obtained directly from the numerical result given by Eq. (6). We see that the analytical formula (20) can give a well description for the disappearance of the density oscillation.

Fig. 6 Displayed is the ratio between $n(x = 0, t_m, s)$ and $n(x = 0, t_m, s = 5)$ for the interacting and non-interacting models.

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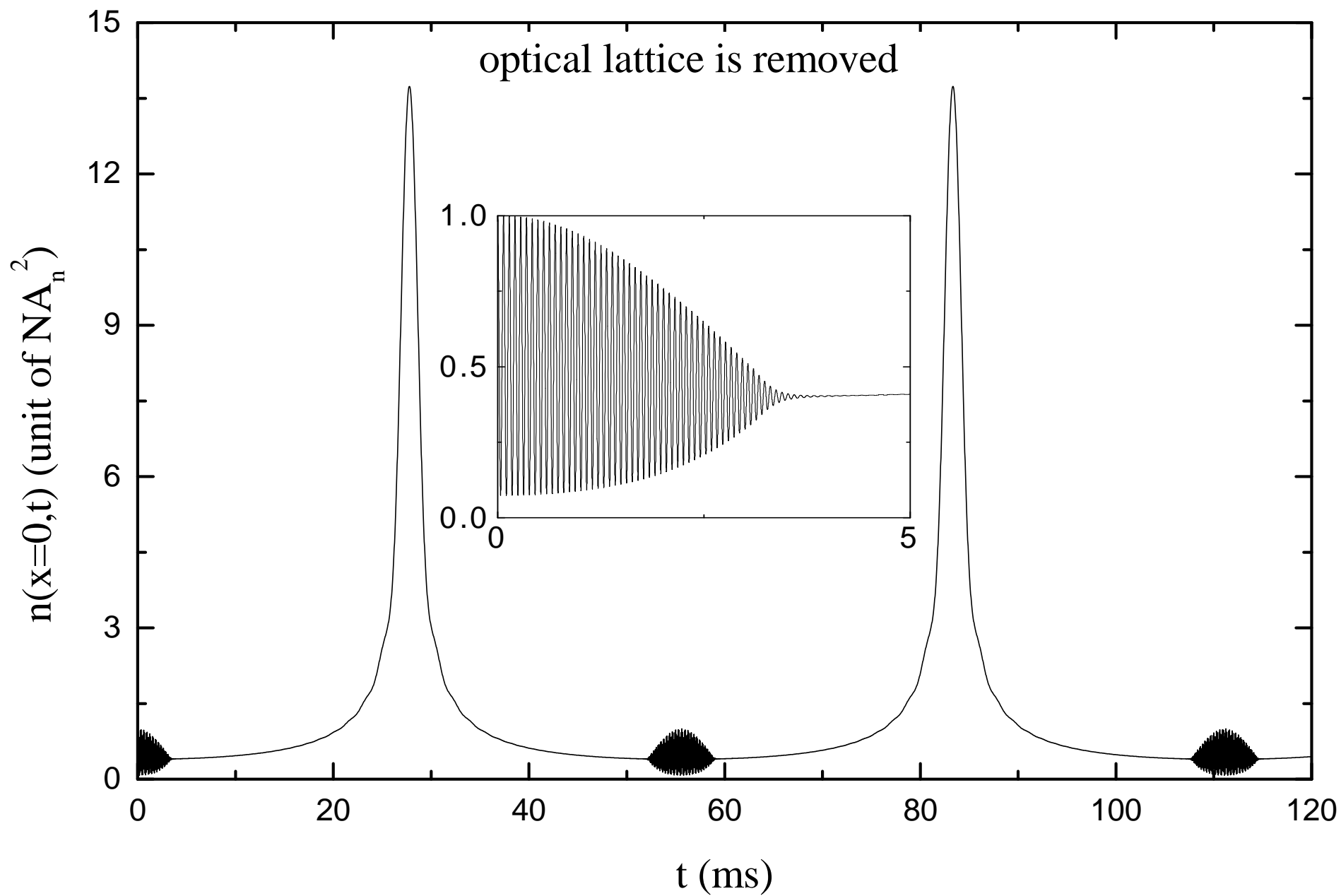


Figure 1 (Xiong et al)

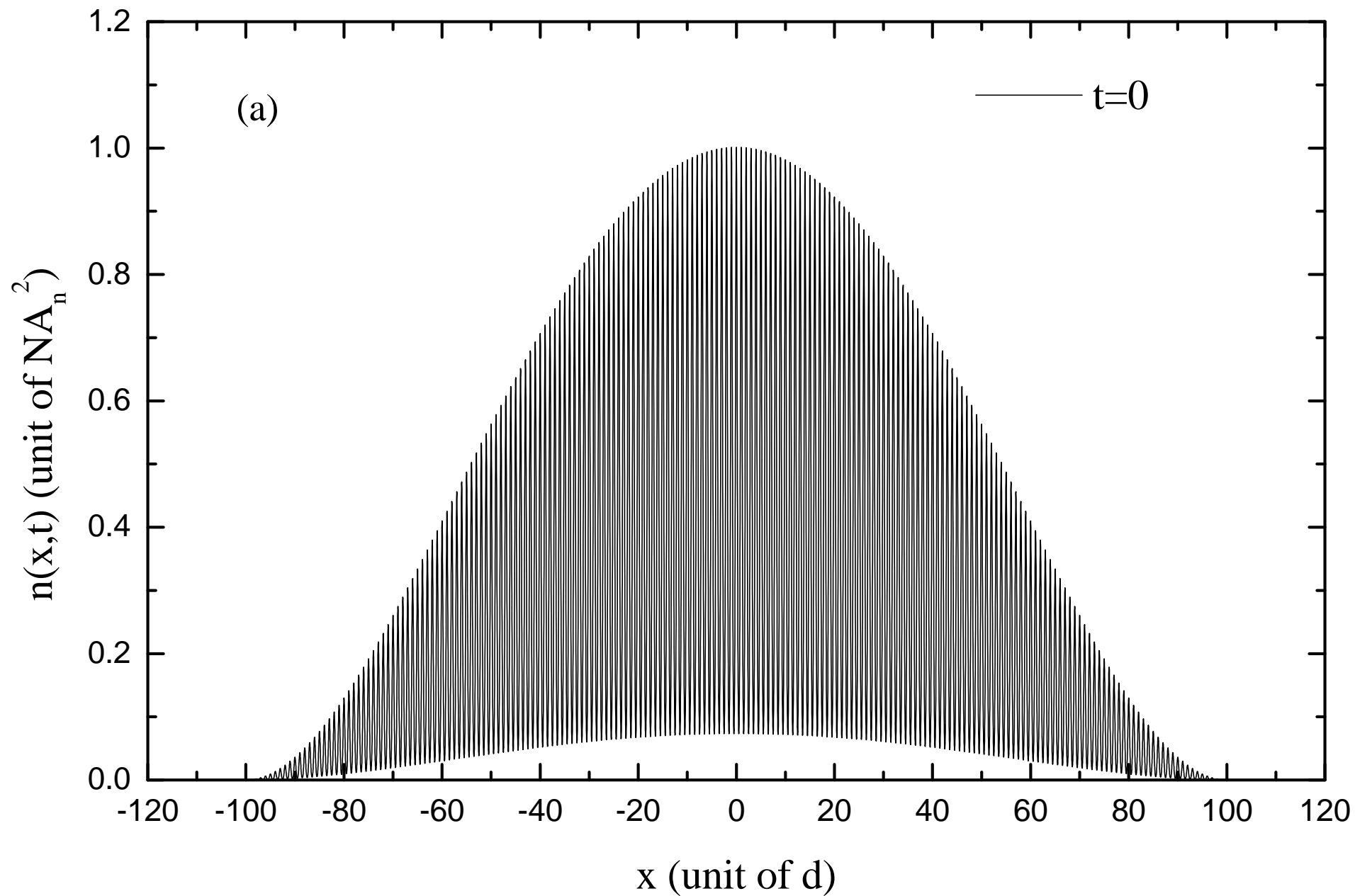


Figure 2a (Xiong et al)

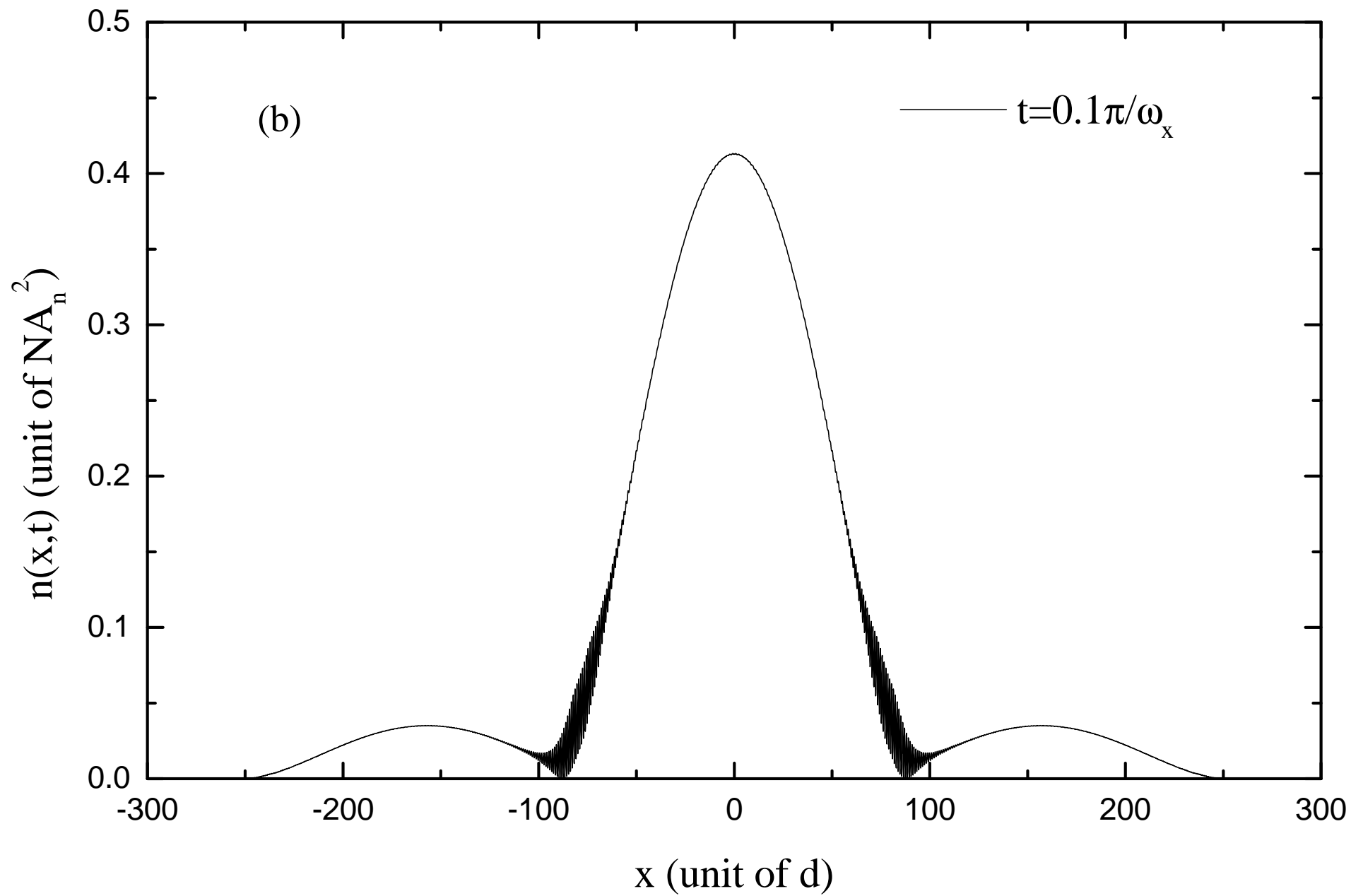


Figure 2b (Xiong et al)

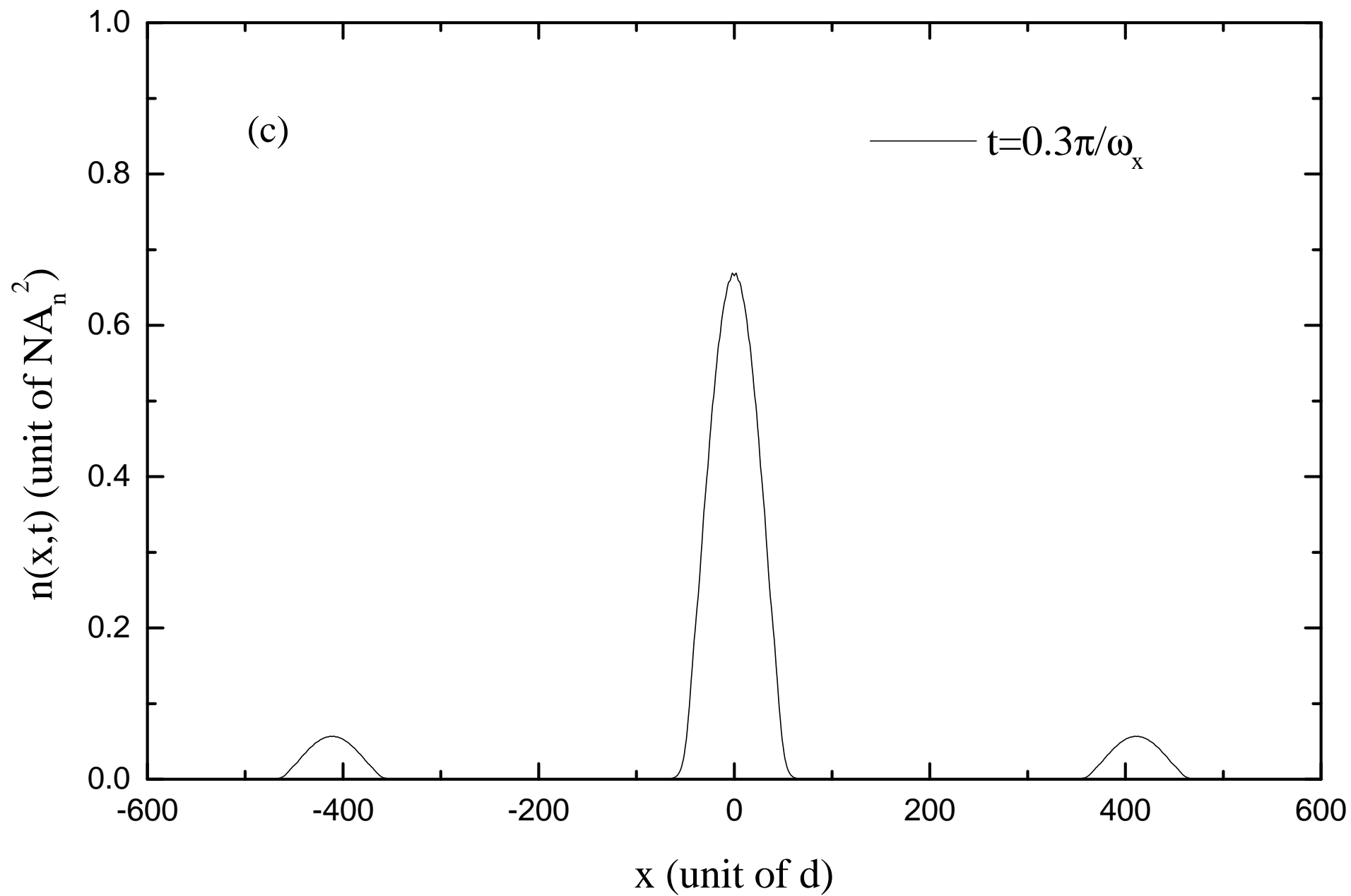


Figure 2c (Xiong et al)

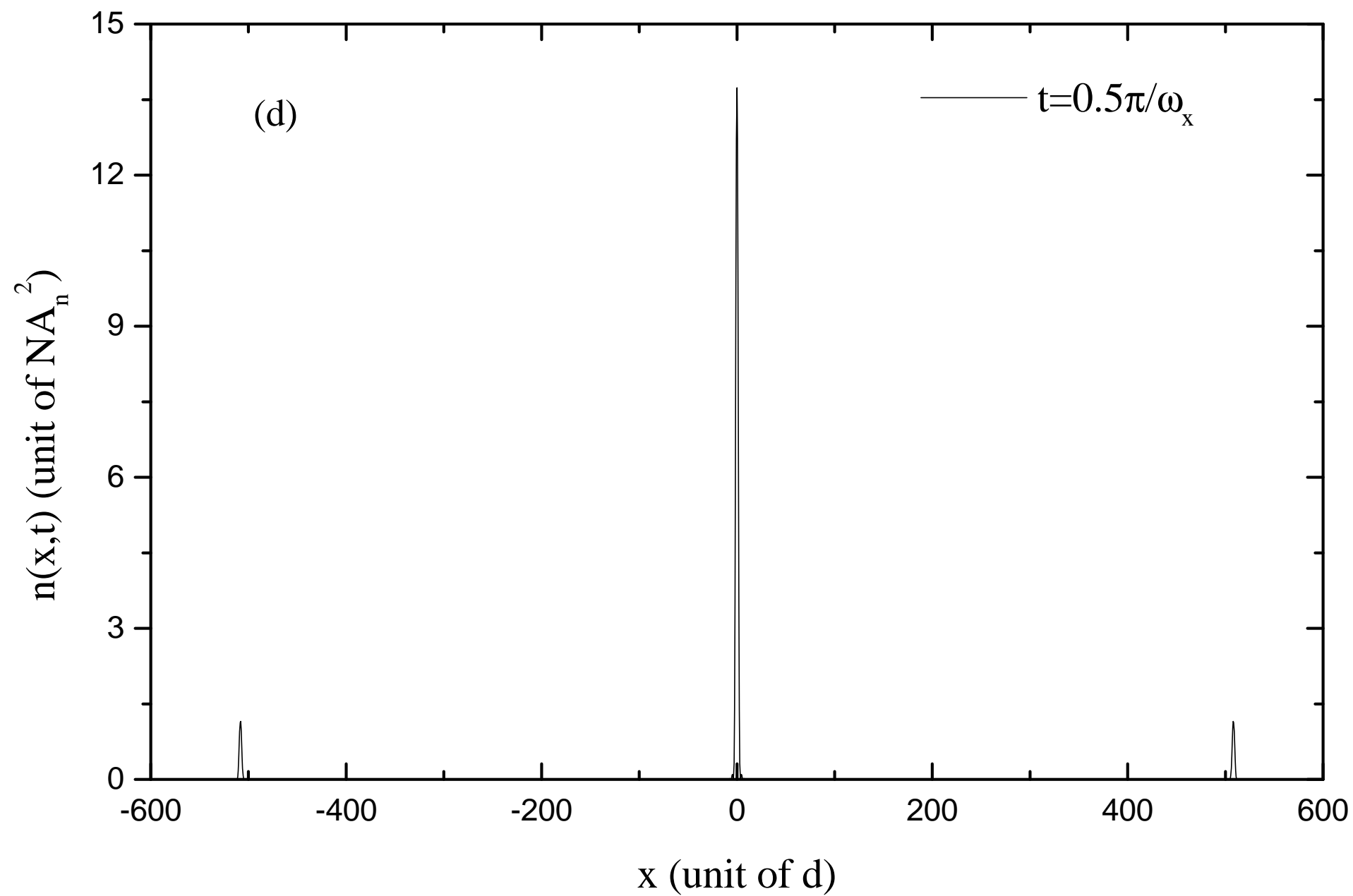


Figure 2d (Xiong et al)

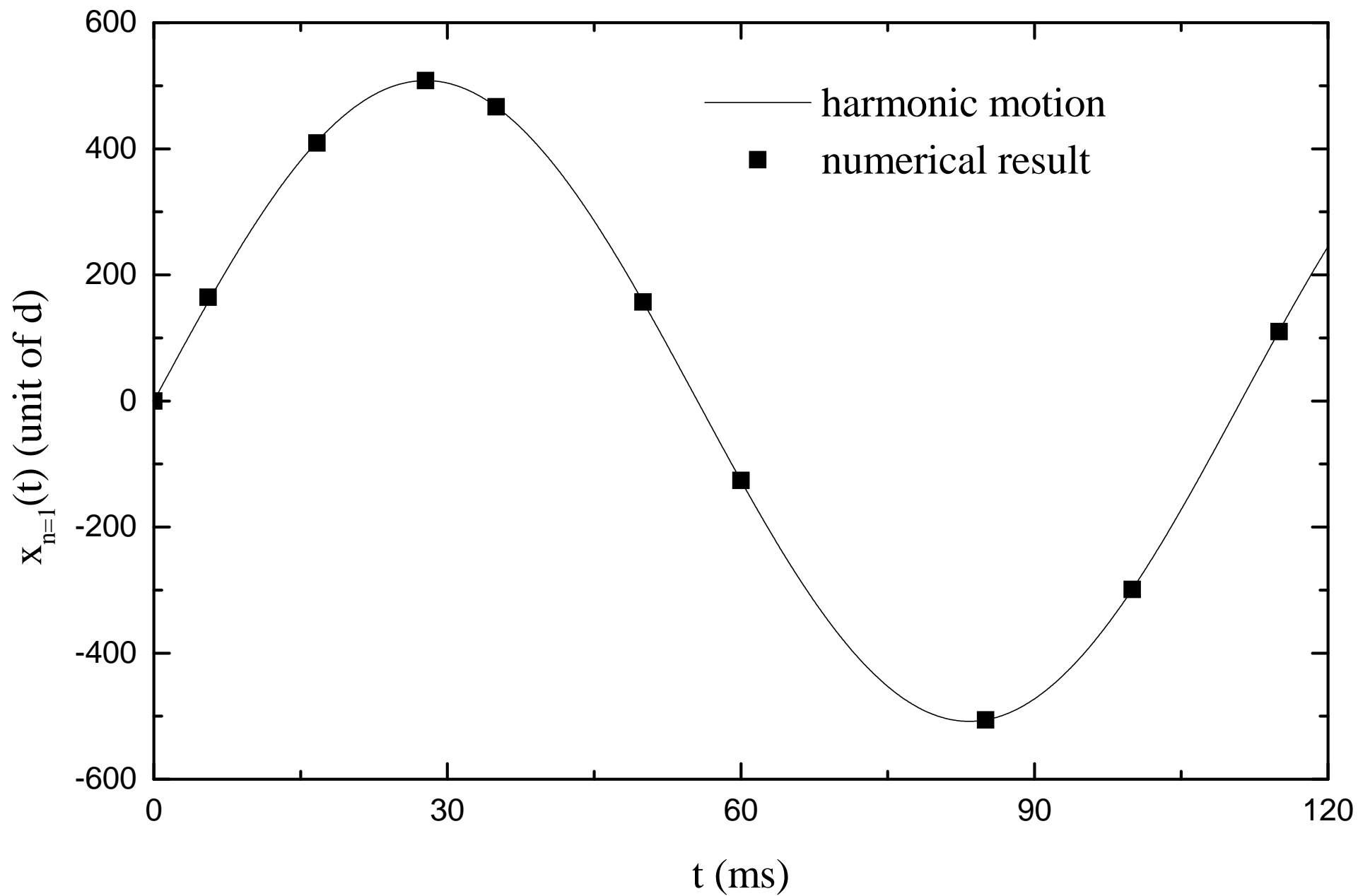


Figure 3 (Xiong et al)

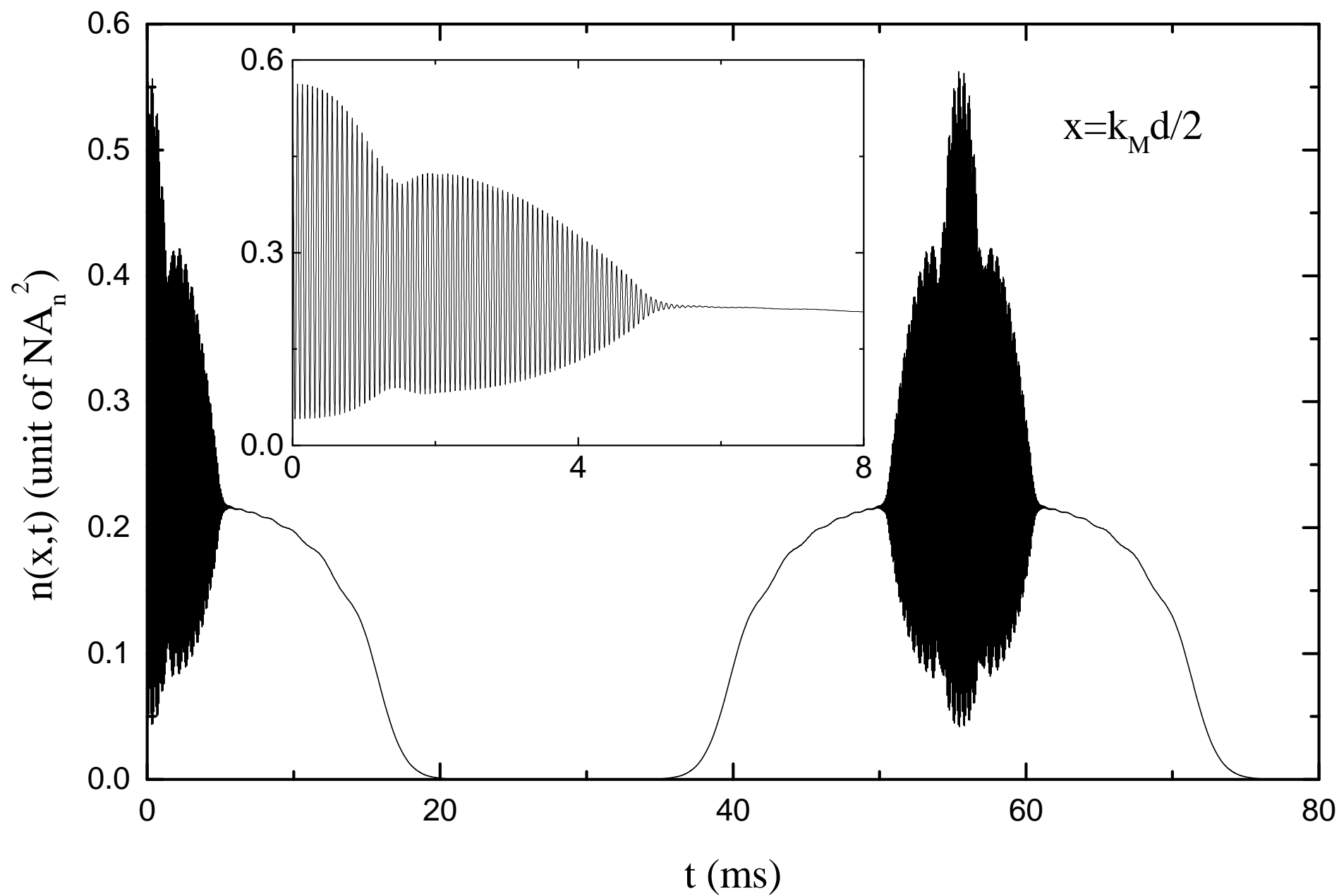


Figure 4 (Xiong et al)

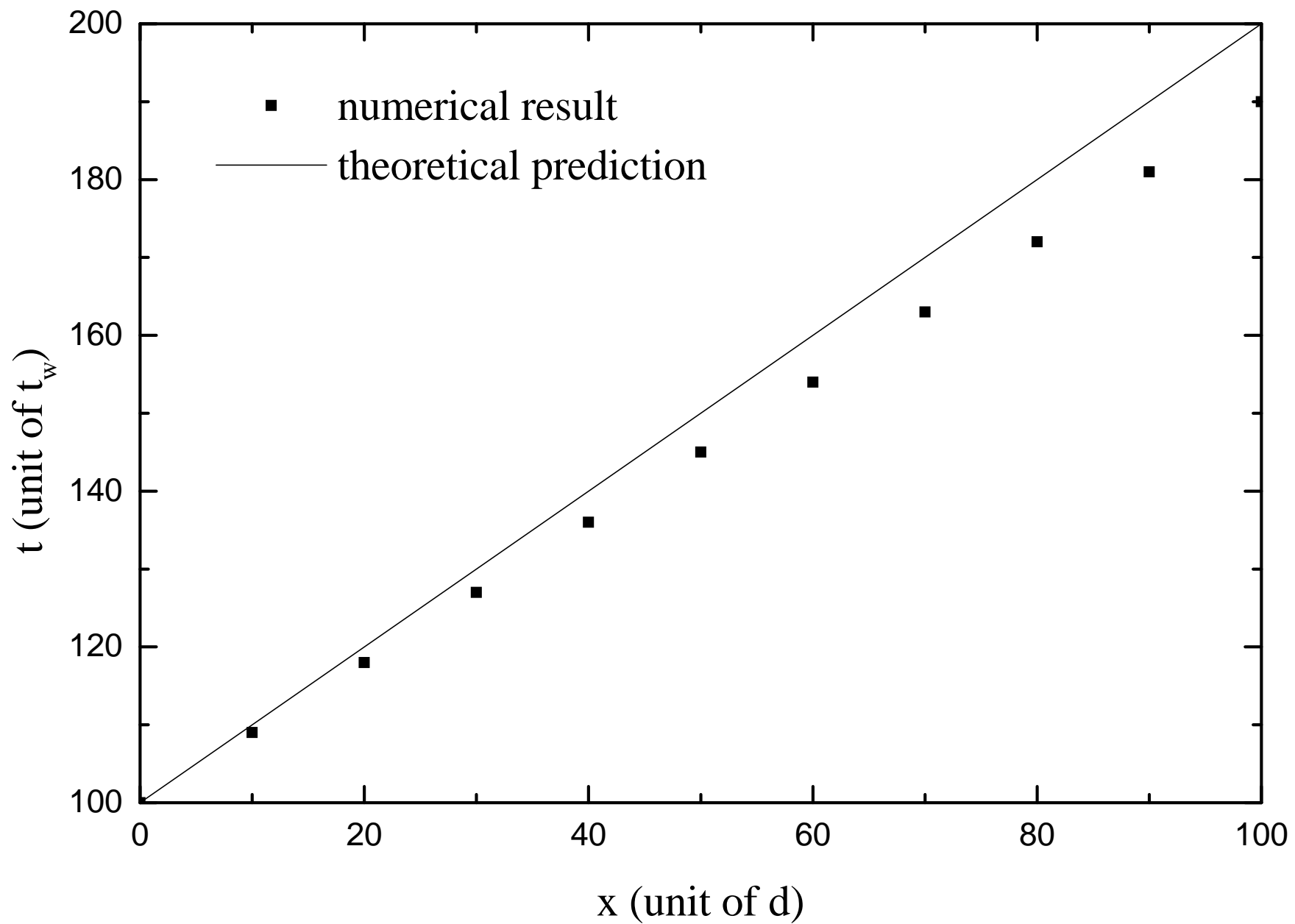


Figure 5 (Xiong et al)

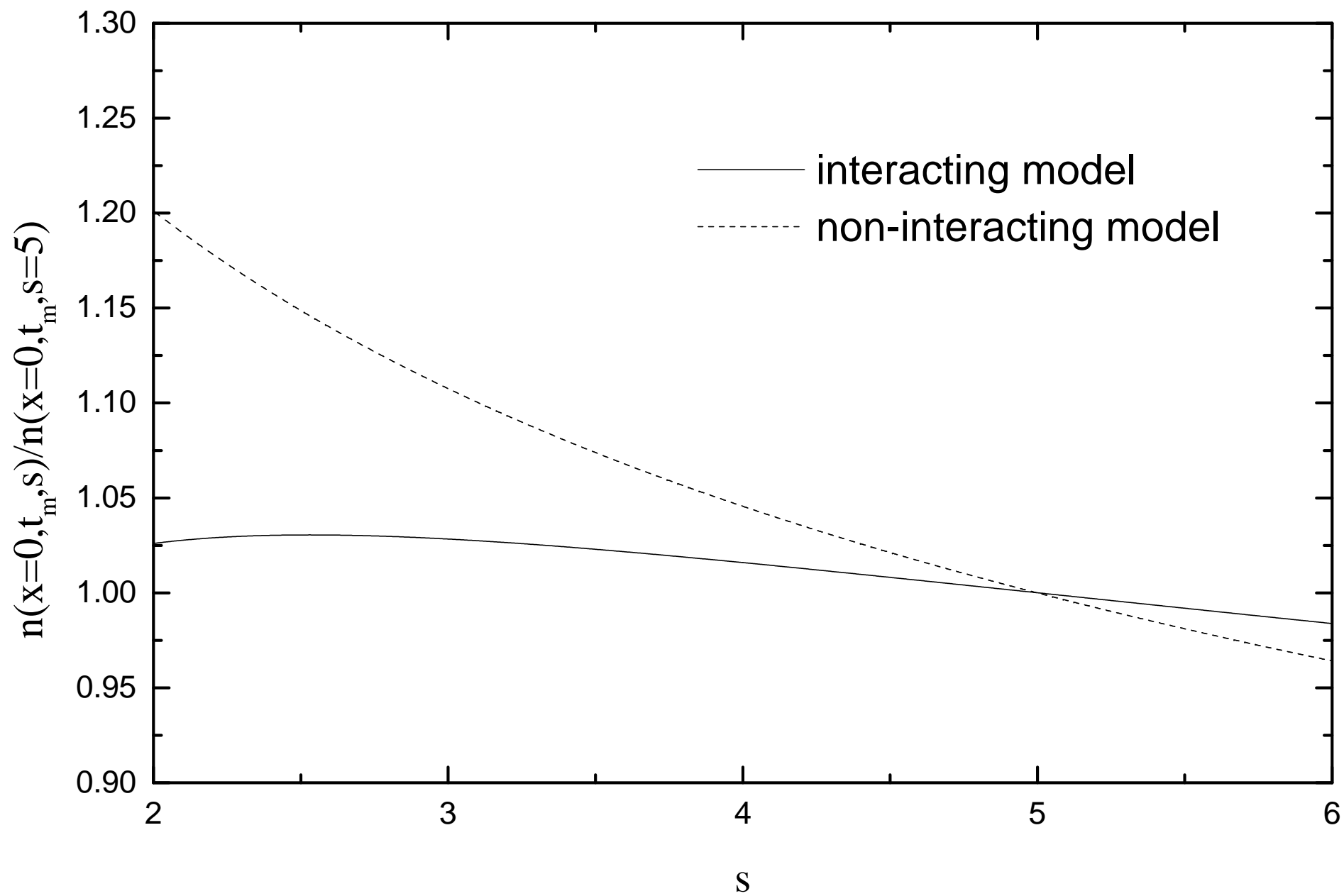


Figure 6 (Xiong et al)